

Total: 30/110 points

A system is described by the following differential equations:

$$\begin{aligned}\dot{x}_1 &= -3x_1 + x_2 + u \\ \dot{x}_2 &= x_1 - 3x_2 + u.\end{aligned}$$

The scalar input to the system is $u(t)$. The two outputs are $y_1(t) = x_1(t)$ and $y_2(t) = x_2(t)$.

- [8 pt] What are the state-space matrices A , B , C , and D that describe this system?
- [6 pt] Is the system overdamped, underdamped or critically damped? Explain your answer.
- [8 pt] Using the state transformation

$$\vec{x}' = \begin{bmatrix} x'_1 \\ x'_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix},$$

we get the transformed state-space system

$$\begin{aligned}\dot{\vec{x}}' &= A'\vec{x}' + B'\vec{u}, \\ \vec{y} &= C'\vec{x}' + D'\vec{u}.\end{aligned}$$

With this choice of transformation, the matrix A' is

$$A' = \begin{bmatrix} -2 & 0 \\ 0 & -4 \end{bmatrix}.$$

Compute the matrices B' , C' and D' .

- [6 pt] Draw the block diagram of your state equations $\dot{\vec{x}}' = A'\vec{x}' + B'\vec{u}$ in the transformed state, \vec{x}' . (Note: your block diagram does not need to show the output equation.)
- [2 pt] In one sentence, comment on our ability to control the state of this system using the given input.

Signals & Systems Question 1

Student ID Number: _____

Signals & Systems Question 1

Student ID Number: _____